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Capturability in a Two-Target "Game of Two Cars"

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The regions of capturability and draw are derived for a two-target pursuit-evasion differential game of two cars. Specifically, using geometric arguments, complete barrier surfaces are constructed that emanate from the boundaries of the usable parts of the targets of the faster and slower players. The targets are, respectively, a sector of a disk and a complete disk. The barrier surface emanating from the latter target provides a solution to the one-target game of two cars, while the regions of capturability in the two-target game of two cars are inferred from the combined effect of both barrier surfaces.

Nomenclature

f	= right-hand side of the kinematic differential equations
n	= normal to the barrier surface
r	= weapon system range
R	= minimum turning radius
R^2	= Euclidian plane
S^1	= one-dimensional thorus
$\mathcal{S}$	= barrier surface
t	= time
T	= target set (terminal manifold)
u	= control variable
w	= speed
x,y	= position of the player in the plane
α	= half-squint angle of the weapon system
β	= maneuverability ratio $\triangleq w_1R_2/w_2R_1$
γ	= speed ratio $\triangleq w_2/w_1$
θ	= heading
τ	= time

Subscripts

$1,2$	= player 1 and player 2, respectively
x,y,θ	= partial derivative with respect to x,y, or θ

I. Introduction

THE literature on two-player qualitative differential games (games of kind) has focused to a large extent on one-target differential games of the pursuit-evasion kind, the game of two cars[1] being a typical example. Admittedly, this formulation is an appealing model if we consider, say, a missile chasing a plane. However, upon considering a dogfight between two planes, ships, or other craft that are both armed and capable of destroying their opponents, it is apparent that the evader, after having employed a suitable evasion tactic (for example, a tight turn), can find himself in the position of a pursuer (i.e., he may find the pursuing craft in front of his guns). In view of these considerations we introduce two-target pursuit-evasion differential games (of kind) of the "game of two cars" type where either player may, depending on the configuration of the players, be the pursuer or the evader. Each target set (T_1 and T_2) is now indicative of the weapon-systems kill (i.e., capturability) capability of the respective player.

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Two-target differential games were introduced in Ref. 2 (see also, Ref. 3). Regions of capturability in the two-target pursuit-evasion differential game of two cars have been obtained in Ref. 4 for target sets in front of each aircraft consisting of a line segment aligned with the velocity vector of the aircraft and problem parameters: a speed ratio of 3/4, maneuverability ratio of 1/3, the weapon system range of the first aircraft is twice its minimum turning radius and the range of the weapon system of the second aircraft is unbounded. In Ref. 5 the target sets T_1 and T_2 are such that the game terminates when either aircraft is directly in front of the other with the heading difference limited to at most 30 and 40 deg, respectively, and with the problem parameters: a speed ratio of 1/2 and a maneuverability ratio of 10/11. In Ref. 6 the target of player 1 is a fan shaped zone in front of 1 while the mirror image of this region below the x axis is taken as T_2, i.e., the winning region of player 2 is rigidly attached to player 1 in the realistic state space. Furthermore, in Ref. 6, for problem parameters: a speed ratio of $\sqrt{2}/2$ and a maneuverability ratio of 1/2, the region of capturability of player 1 is obtained when player 1 acts as the pursuer and player 2 is the evader. In addition, the region of capturability (in fact reachability) for player 2 is obtained upon letting player 2 act as the pursuer under the assumption that player 1 does not maneuver.

In the present paper we consider the two-target game of two cars with a fan-shaped target set T_1 for player 1 and a circular target set for player 2, and parameter values: a speed ratio of 1/2 and a maneuverability ratio of 1. The reader is also referred to Sec. II where the target sets are described in a concise manner.

In this respect the present paper is a sequel to Ref. 7 where the corresponding homicidal chauffeur version (the minimum turning radius of player 2 is zero so that the reduced space is two dimensional) of the two-target game of two cars has been analyzed.

Instead of employing the value function approach of Isaacs (Hamilton-Jacobi partial differential equation and related costate functions) we have focused our approach on the geometric construction of the complete barrier surfaces (in the reduced state space) which emanate from the boundary of the usable parts of the targets T_1 and T_2.

These barrier surfaces are constructed employing (geometrical) invariance arguments, as elaborated on in Ref. 8, and include the dispersal lines on the barrier surfaces and the curves that delimit these surfaces. The solution to the two-target two-car game of kind is then arrived at by discarding the appropriate part of one of the T_2 surfaces beyond the curve along which the two barrier surfaces intersect, as a result of which the state space is partitioned into regions from which either both players, only one of them, or neither can force a win. Consequently the pursuer/evader roles for players 1 and 2 are determined in these regions.

Specifically, following a statement of the problem in Sec. II, in Sec. III we outline the general geometric method of construction of the barrier surfaces introduced in Ref. 8. In Sec. IV we present the barrier surface that emanates from the target set T_1, while in Sec. V we present the barrier surface that emanates from the target set T_2. The latter case provides a solution to the pursuit-evasion differential game of two cars from the point of view of player 2; namely, player 2 with the target set T_2 is the pursuer and player 1 is the evader. Furthermore, since T_2 is circular and player 2 is slower than player 1, the barrier presented in Sec. V provides a solution to the research problem posed by Isaacs[1] of pursuit-evasion in the game of two cars where the pursuer is slower than the evader.

The barrier presented in Sec. IV does not, however, provide a solution to a one-target game of two cars (player 1 is the pursuer, endowed with a target set T_1, and player 2 is the evader) since the boundary condition for the barrier surface is not the natural boundary condition but is imposed by the geometry of the two-target game.

Finally in Sec. VI we combine the barriers presented in Secs. IV and V to provide the game-of-kind (capturability) solution to the two-target pursuit-evasion differential game of two cars (dogfight) just posed.

II. Mathematical Modeling

We consider the following model for a dogfight in the plane, the model being an extension of that evolved for the pursuit-evasion differential game of two cars.[1] Two points, i.e., "players," 1 and 2, whose positions at any instant are given by (x_1,y_1) and (x_2,y_2), move in a plane at constant speed w_i in the direction θ_i, where θ_i is an angle measured from the positive y_i axis in a clockwise direction, $i=1,2$.

The maneuverability of the players is determined by their minimum turning radii R_1 and R_2, respectively, and they steer by selecting at each instant the value of the curvature of their trajectories by choosing an appropriate value of the controls $u_1 \in [-1,1]$ and $u_2 \in [-1,1]$, respectively, which curvature then determines their actual instantaneous turning radius.

Suppose that the weapon system of player 1 has a range r_1 and a half-squint-angle α_1 and the weapon system of player 2 has an all-aspect range of r_2 (i.e., player 2 has a swivelling gun). Let T_i denote the (closed) set of state space points for which the weapons of player i, $i=1,2$, are effective.

The objective of each player is to force his (uncooperative) opponent into his target, while at the same time he avoids entering his opponent's target. Formally we say that player 1 scores a win if there exist $t' \geq 0$ and $\tau > 0$ such that $(x_2(t), y_2(t)) \in T_1$ for $t \in [t', t'+\tau)$ and $(x_1(t), y_1(t)) \notin T_2$ on any open subinterval of $[0, t'+\tau)$; similarly for player 2.

Thus a player can always capture his opponent in the interior of his target, but can capture him on the boundary of his target only if his opponent is there for a nonzero period of time. This two-target game of two cars is illustrated in Fig. 1 and is a special case (T_2 is circular) of the more general setting presented in Ref. 7.

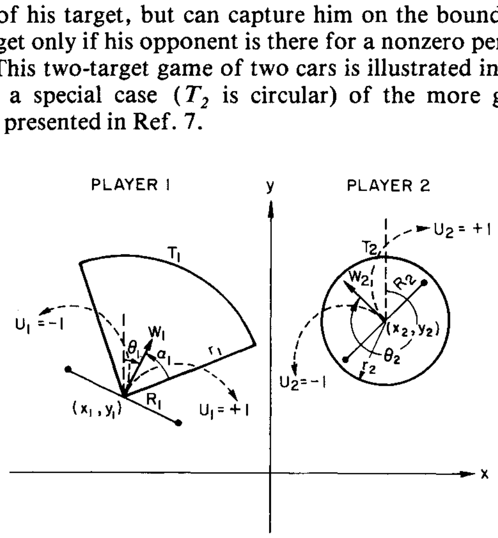

Fig. 1 The two-target pursuit-evasion game in the plane.

By fixing the origin of the coordinate system (x,y) at the position occupied by player 1, aligning the y axis with the direction of motion of player 1, and considering the relative motion of players 1 and 2, the state space of the game is reduced to $R^2 \times S^1$: the state vector $(x,y,\theta)'$ designates the position (x,y) of player 2 in the plane and the angle θ is the velocity vector of player 2 measured clockwise from the positive y axis (the direction in which player 2 is heading). When the equations of motion have been transformed to dimensionless form [i.e., when $x/R_1 \to x$, $y/R_1 \to y$, $r_1/R_1 \to r_1$, $r_2/R_1 \to r_2$, $tw_1/R_1 \to t$ and $\gamma \triangleq w_2/w_1$, $\beta \triangleq (1/\gamma)R_2/R_1$], they become

$$\frac{dx}{dt} = -yu_1 + \gamma\sin\theta \qquad x(0) = x_0$$

$$\frac{dy}{dt} = xu_1 - 1 + \gamma\cos\theta \qquad y(0) = y_0$$

$$\frac{d\theta}{dt} = -u_1 + \frac{u_2}{\beta} \qquad \theta(0) = \theta_0$$

$$(t \geq 0, \quad |u_1| \leq 1, \quad |u_2| \leq 1) \tag{1}$$

and the parameters of the specified capture sets then imply the cylindrical target sets

$$T_1 = \{(x,y,\theta) \mid x^2 + y^2 \leq r_1^2, \quad 90 \text{ deg} - \alpha_1 \leq \arctan(y/x) \leq 90 \text{ deg} + \alpha_1, \quad y \geq 0\} \tag{2}$$

and

$$T_2 = \{(x,y,\theta) \mid x^2 + y^2 \leq r_2^2\} \tag{3}$$

In this notation the target sets were

$$T_i = \{(x,y,\theta) \mid x=0, \; 0 \leq y \leq r_i\} \qquad (i=1,2)$$

$$T_1 = \{(x,y,\theta) \mid x=0, \; y \geq 0, \; -30 \text{ deg} \leq \theta \leq 30 \text{ deg}\}$$

$$T_2 = \{(x,y,\theta) \mid x = y\tan\theta, \; y \leq 0, \; -40 \text{ deg} \leq \theta \leq 40 \text{ deg}\}$$

and

$$T_1 = \{(x,y,\theta) \mid x^2 + y^2 \leq 1/8, \; 60 \text{ deg} \leq \arctan(y/x) \leq 120 \text{ deg}, \; y \geq \sqrt{2}/80\}$$

$$T_2 = \{(x,y,\theta) \mid x^2 + y^2 \leq 1/8, \; 60 \text{ deg} \leq \arctan(-y/x) \leq 120 \text{ deg}, \; y \geq -\sqrt{2}/80\}$$

in Refs. 4-6, respectively.

Note that now (in the reduced state space) the target T_2 is attached to the origin instead of the point (x,y), i.e., when the point (x,y) reaches the boundary of the target set T_2, player 1 comes into the range of the weapons of player 2. The object of player i is thus to get the point (x,y) into T_i without passing through his opponent's target. The game in this reduced state space is illustrated in Fig. 2, and the targets, T_i, $i=1,2$, as they appear in the reduced state space are illustrated in Fig. 3.

III. Construction of Barrier Surfaces—An Outline

The targets T_1 and T_2 as they would appear in $R^2 \times [0,2\pi]$ (note that $\theta=0$ and $\theta=2\pi$ are identified and that the actual state space is $R^2 \times S^1$) are illustrated in Fig. 3, and the boundary of their usable parts[1] (BUP) on the target surfaces

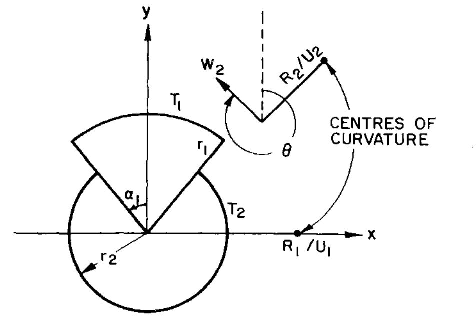

Fig. 2 The game illustrated in Fig. 1 in the reduced state space.

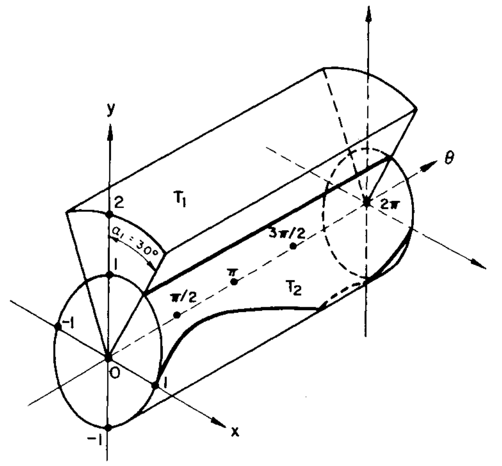

Fig. 3 The targets T_1 and T_2 in $R^2 \times [0,2\pi]$.

can be evaluated to obtain

Boundary of usable part of T_1

$$x_0(\theta_0) = 1/2 \qquad y_0(\theta_0) = \sqrt{3}/2 \tag{4}$$

Boundary of usable part of T_2

$$x_0(\theta_0) = (2-\cos\theta_0)/\sqrt{5-4\cos\theta_0}$$

$$y_0(\theta_0) = \sin\theta_0/\sqrt{5-4\cos\theta_0} \tag{5}$$

The boundary of the usable part of T_1 is in our case imposed by T_2. If, however, the radius of T_2 were less than unity, then part or all of the natural boundary of the usable part of T_2 would emerge and in these regions would be given by

$$x_0(\theta_0) = [\sin(\theta_0 - 30\ \text{deg}) + 1]/4$$

$$y_0(\theta_0) = [\sin(\theta_0 - 30\ \text{deg}) + 1]\sqrt{3}/4 \tag{6}$$

The barrier surface is a semipermeable surface[1] that emanates from the boundary of the usable part of the target set (T_1 or T_2). Indeed, a semipermeable surface $\mathcal{S}$ is a surface on which the saddle-point condition

$$\max_{|u_i| \le 1}\ \min_{|u_j| \le 1}\ (n,f) = 0 \qquad (i \ne j;\quad i,j = 1,2) \tag{7}$$

holds, where n is a vector normal to the surface $\mathcal{S}$, f denotes the right-hand side of Eq. (1) and (n,f) the inner product of the vectors n and f. If $\mathcal{S}$ is described by a function $y(x,\theta)$ then elementary differential geometry tells us that $n = (-y_x,\ 1,\ -y_\theta)$ (here $y_x \triangleq \partial y/\partial x$, $y_\theta \triangleq \partial y/\partial \theta$) and Eq. (7), for the case $i=1$, $j=2$, implies that (see Ref. 8) $\mathcal{S}$ must satisfy the partial differential equation

$$(x + yy_x + y_\theta)u_1 - y_\theta u_2 - y_x(\sin\theta)/2 + (\cos\theta)/2 - 1 = 0 \tag{8}$$

where

$$u_1 = \text{sign}(x + yy_x + y_\theta) \qquad u_2 = \text{sign}(y_\theta) \tag{9}$$

The Eqs. (9) indicate that we must consider the controls $|u_i| = 1$, $i = 1,2$ in which case either $u_1 = u_2 = \pm 1$ or $u_1 = -u_2 = \pm 1$. The case $u_1 = \pm 1$ with $u_2 = 0$ is also important, as it is used to construct the universal curve which separates regions in which $u_1 = u_2$ and $u_1 = -u_2$.

For given u_1 and u_2 the equations of motion Eq. (1) are then linear and have the following solutions:

For $u_1 = -u_2$

$$x(\theta,\theta_0) = \left[x_0(\theta_0) - \frac{1}{u_1}\right]\cos\left(\frac{\theta-\theta_0}{2}\right) + \left[2 + \cos\theta - \cos\left(\frac{\theta+\theta_0}{2}\right)\right]\Big/2u_1 + y_0(\theta_0)\sin\left(\frac{\theta-\theta_0}{2}\right)$$

$$y(\theta,\theta_0) = \left[\frac{1}{u_1} - x_0(\theta_0)\right]\sin\left(\frac{\theta-\theta_0}{2}\right) + \left[\sin\left(\frac{\theta+\theta_0}{2}\right) - \sin\theta_0\right]\Big/2u_1 + y_0(\theta_0)\cos\left(\frac{\theta-\theta_0}{2}\right) \tag{10}$$

For $u_1 = u_2$

$$[yu_1 - (\sin\theta_0)/2]^2 + [xu_1 - 1 + (\cos\theta_0)/2]^2 = C(\theta_0) \tag{11}$$

where $C(\theta_0)$ is evaluated from the left-hand side of Eq. (11) at $x = x_0(\theta_0)$, $y = y_0(\theta_0)$.

For $u_2 = 0$

$$x(\theta,\theta_0) = [x_0(\theta_0) - 1/u_1]\cos(\theta-\theta_0) + [2 - (\theta-\theta_0)\sin\theta]/2u_1 + y_0(\theta_0)\sin(\theta-\theta_0)$$

$$y(\theta,\theta_0) = [1/u_1 - x_0(\theta_0)]\sin(\theta-\theta_0) - (\theta-\theta_0)\cos(\theta)/2u_1 + y_0(\theta_0)\cos(\theta-\theta_0) \tag{12}$$

By construction, the characteristics[9] of a surface $\mathcal{S}$ satisfying Eq. (8) and emanating from the BUP are solutions to Eq. (1) and therefore satisfy Eqs. (10-12). In Ref. 8 it is shown in detail how the surface $\mathcal{S}$, satisfying Eqs. (8) and (9) and emanating from the BUP can be constructed using Eqs. (10-12). Basically, from the BUP of the target, surfaces are constructed using Eqs. (10) and (11) with nominal extreme values for u_1 and u_2 (viz., $u_i = \pm 1$, $i = 1,2$). Only these parts of the surfaces compatible with Eqs. (9) that are geometrically significant (e.g., external to target) are retained. The result is two bounded surfaces which are unconnected away from the target but can be patched together by inserting a universal curve (UC) at the point of connection between the two surfaces on the target and using the UC as the new boundary condition from which new surfaces satisfying Eqs. (8) and (9) can be generated; the UC is given by Eqs. (12). The result is a surface made up of four parts, connected in sequence, viz: 1) $u_1 = u_2$ and the surface emanates from the BUP; 2) $u_1 = u_2$ and the surface emanates from the UC; 3) $u_1 = -u_2$ and the surface emanates from the UC; and 4) $u_1 = -u_2$ and the surface emanates from the BUP. Note that in all four cases the value of u_1 is the same and is either $+1$ or -1 depending on whether the surface emanates from T_1 or T_2, respectively.

The barrier surface is delimited by the right-hand sides of the equations in Eqs. (9) taking on the value 0.

In fact, at this stage it must be verified whether it is not possible to continue the surface $\mathcal{S}$, beyond this point, with $u_1 = -1$ [and the right-hand side of the first of Eqs. (9) thereafter assuming non-positive values], thus producing the situation depicted in Fig. 3, cases 5-7, in Ref. 4. It turns out, however, that for our specific problem parameters this is indeed not the case. The reader is also referred to Sec. 3 of Ref. 7, where this point is further elaborated on.

In addition, the barrier surface is periodic, with period 2π (recall that our reduced state space is $R^2 \times S^1$). Therefore the section of the barrier beyond 2π overlaps with the barrier in the region $\theta > 0$ and the line of intersection gives rise to the dispersal curve. In addition, the left- and right-hand barriers are related by $y(x,\theta) = y(-x, 2\pi - \theta)$.

Finally, the reader is also referred to Ref. 8 where a detailed method is given for constructing the barrier in the pursuit-evasion differential game of two cars.

We also remark here that, although strictly speaking Eq. (7) is a necessary condition, the reader can readily convince himself (see also Ref. 7) that this is immaterial as far as the construction of the barrier surface is concerned.

IV. Barrier Surface Emanating from T_1

The barrier surface in the pursuit-evasion differential game of two cars when player 1, endowed with the target set T_1, is the pursuer and player 2 is the evader, is constructed along the lines of Sec. II. The barrier is open and player 1 has full state space capturability. The barrier surface emanates from the T_1 BUP [recall that the BUP of T_1, given in Eq. (4), is imposed by T_2 and is not the natural boundary condition Eq. (6)] and eventually folds under itself as projected on the (x,θ) plane in Fig. 4. The four parts described in Sec. III are also indicated. The three trajectories emanating from B in Fig. 4 with $u_1 = +1$ and $u_2 = -1, 0, +1$ pass over the envelope ($|y_x| = \infty$) at G, H, and J, respectively, and terminate at F, N and M, respectively. Actually both sets of criteria given by the Eqs. (9) are used to terminate the surface, thus yielding the barrier boundary. Indeed, the barrier is terminated at $y_\theta = 0$ by the lines AL and CD and by $x + yy_x + y_\theta = 0$ on the line DEFNKL; on the portion EFNK of this line the barrier terminates after it has already turned back toward the plane $(x=0, y, \theta)$. In addition, since the barrier surface is periodic with period 2π we superimpose on Fig. 4 a version of Fig. 4, translated by 2π, thus obtaining the dispersal curve, as the line of intersection of two surface sheets, which line emanates from 0 and 360 deg. Hence the cross sections (in the (x,y) plane—see Fig. 5) taken at 30, 60, 90 and 120 deg also contain the cross sections taken at 390, 420, 450 and 480 deg and the latter cross sections of the surface are shown in the former cross sections in Fig. 5 by means of broken lines.

These parts (beyond the dispersal curve) play a role in determining the complete (open) barrier surface which is needed to construct the region in which a swerve maneuver is employed by player 1 in order to effect capture of player 2. Indeed, from points below and close to the barrier surface player 1 will play $u_1 = -1$ when $x > 0$ and $u_1 = +1$ when $x < 0$ thereby moving away from player 2 until a switching surface is reached and the strategy of player 1 is reversed. This switching surface is joined on to the end line of the barrier surface, but its actual form will depend on the particular game of degree being considered, e.g., minimum time to capture. The capturability (game-of-kind) solution is thus a prerequisite to finding a solution to any game of degree whose solution then consists of deriving the appropriate switching surfaces. The swerve maneuver is discussed in detail in Ref. 7 in the special case where player 2 has infinite maneuverability, i.e., $R_2 = 0$ (see also, Fig. 6).

V. Pursuit-Evasion Game: Player 2 Pursuer and Player 1 Evader

The barrier surface in the pursuit-evasion differential game of two cars when player 2, endowed with the target set T_2, is the pursuer and player 1 is the evader, is constructed along lines similar to those of Sec. III. The barrier is closed, and player 2 can capture in the region enclosed between the barrier surface and the target set T_2; in the rest of the state space player 1 can escape. Specifically, the barrier surface emanates from the T_2 BUP and eventually the right-hand and left-hand sides of the barrier surfaces meet, along the line MN as projected on the (y,θ) plane in Fig. 7.

Furthermore, in Fig. 7 parts of the barrier beyond the dispersal curve are not indicated since the barrier is closed (i.e., the left- and right-hand barriers meet—see the cross sections in Fig. 8) and these parts play no role in demarcating the region of capturability.

This situation contrasts with that of Fig. 4 (see also Sec. III) where the parts of the barrier beyond the dispersal curve are illustrated, as in this case the left- and right-hand barriers do not meet, and these parts play a role in determining the complete barrier surface. Thus, the barrier associated with T_2 is perfectly matched by the dispersal lines emanating from 60 and 420 deg (see Fig. 7).

Figures 7 and 8 provide a detailed illustration of the closed barrier in the pursuit-evasion game of two cars, when the slower player (player 2) is the pursuer and the faster player (player 1) is the evader. The reader is referred to Ref. 1, p. 244, where pursuit by a slower player is posed as a research problem. The solution here has been facilitated by fixing the coordinate axes with respect to the faster player instead of the pursuer. Hence when the results of Fig. 8 are used, the pursuer must transform back to a set of coordinates attached to his position.

VI. Two-Target Pursuit-Evasion Game

The results of Secs. IV and V are combined to yield the regions of capturability for either player. Indeed, the cross sections in Fig. 5 provide a complete solution to the problem

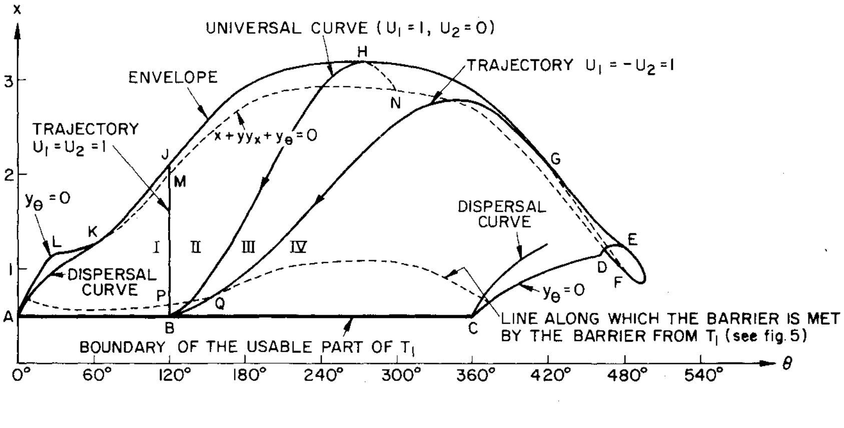

Fig. 4 Projection of the barrier surface from T_1 onto the (x,θ) plane.

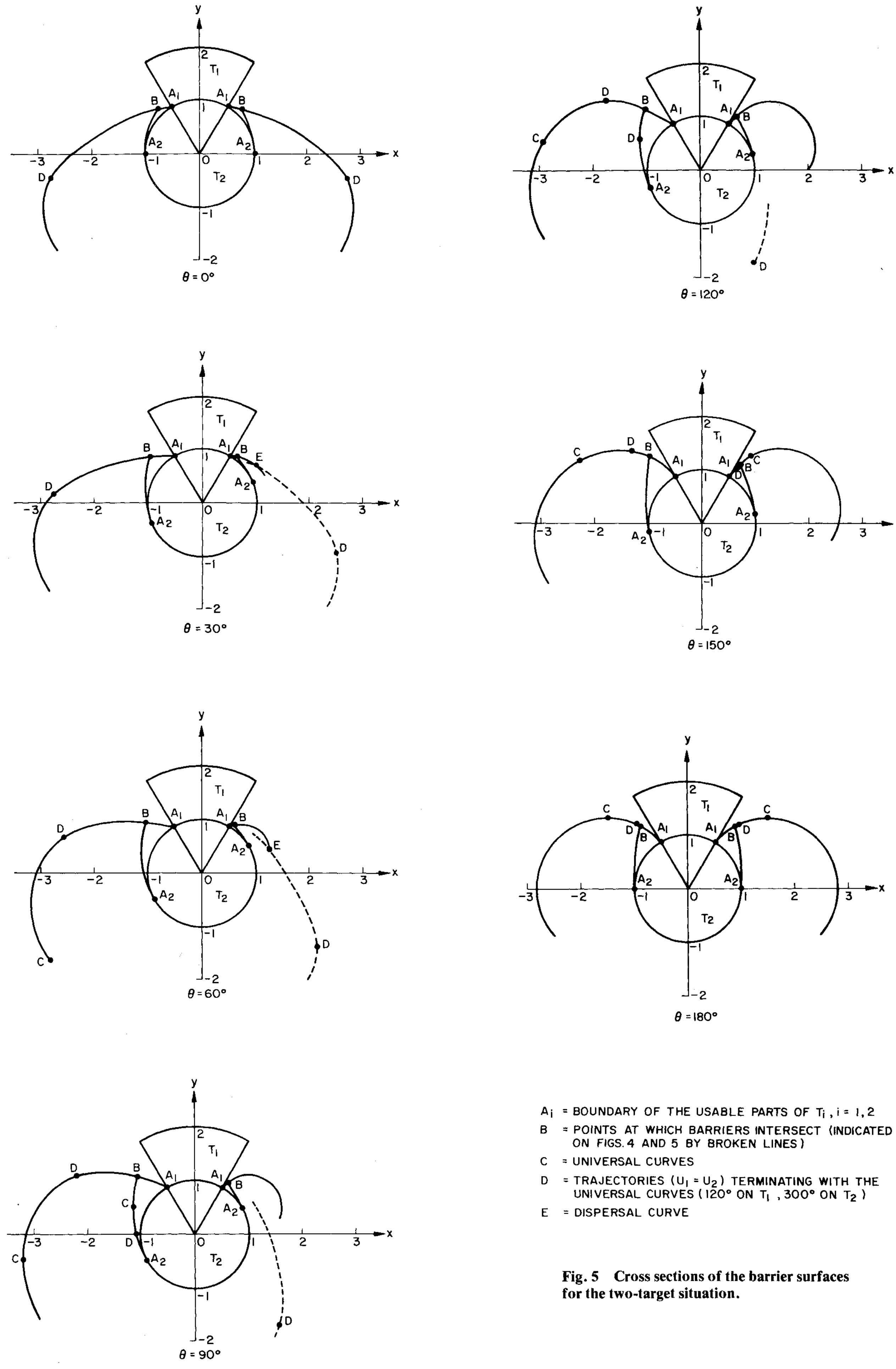

Fig. 5 Cross sections of the barrier surfaces for the two-target situation.

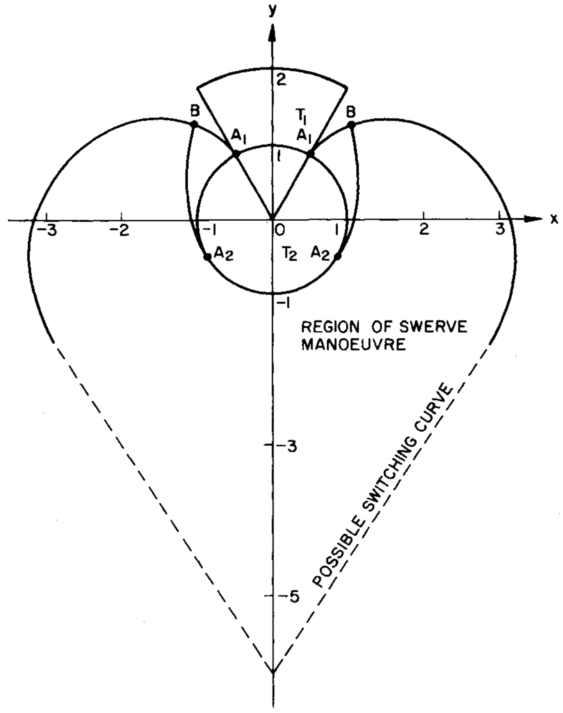

Fig. 6 Solution to the homicidal chauffeur version of the two-target game of two cars presented in Fig. 7 (player 2 has infinite maneuverability).

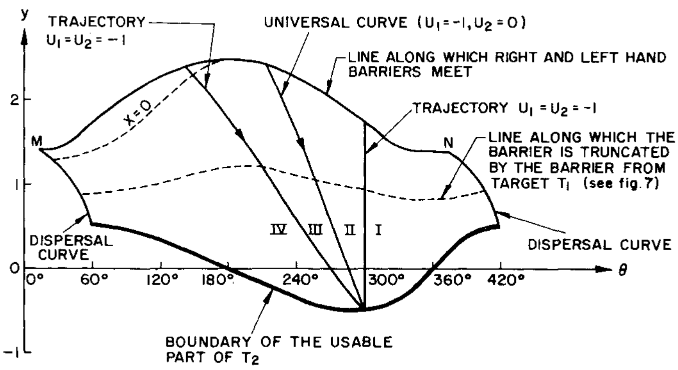

Fig. 7 Projection of the barrier surface from T_2 in the absence of T_1 onto the (y,θ) plane.

of capturability in the two-target pursuit-evasion differential game of two cars (dogfight).

Specifically, the interior of the two regions ($x>0$ and $x<0$) bounded in cross section (see Fig. 5) by the lines A_1B, BA_2, A_2A_1 represents the set of all initial points from which player 2 can force a win, while all points exterior to these regions and excluding T_2 and the actual barrier surfaces are points from which player 1 can force a win. The points on the barrier surface which emanate from T_1, namely the segments A_1B in Fig. 5 (or the region between the BUP and the broken line in Figs. 4 and 7), are initial conditions which result in a draw (a simultaneous win for both players); the exception is the region PBQP (see Fig. 4) on the barrier surface, but not containing the points between P and Q, which is a region of win for player 1. This follows from Eq. (6) and the definition of a win in Sec. I. In the same way, the points on the barrier surface which emanate from T_2, namely the segments A_2B in Fig. 5, are initial conditions which result in a win for player 2, since player 1 only grazes the second player's target set.

For the sake of comparison we present in Fig. 6 the regions of capturability in the corresponding two-target homicidal chauffeur ($R_2=0$) version of this game.[7] It is evident that the

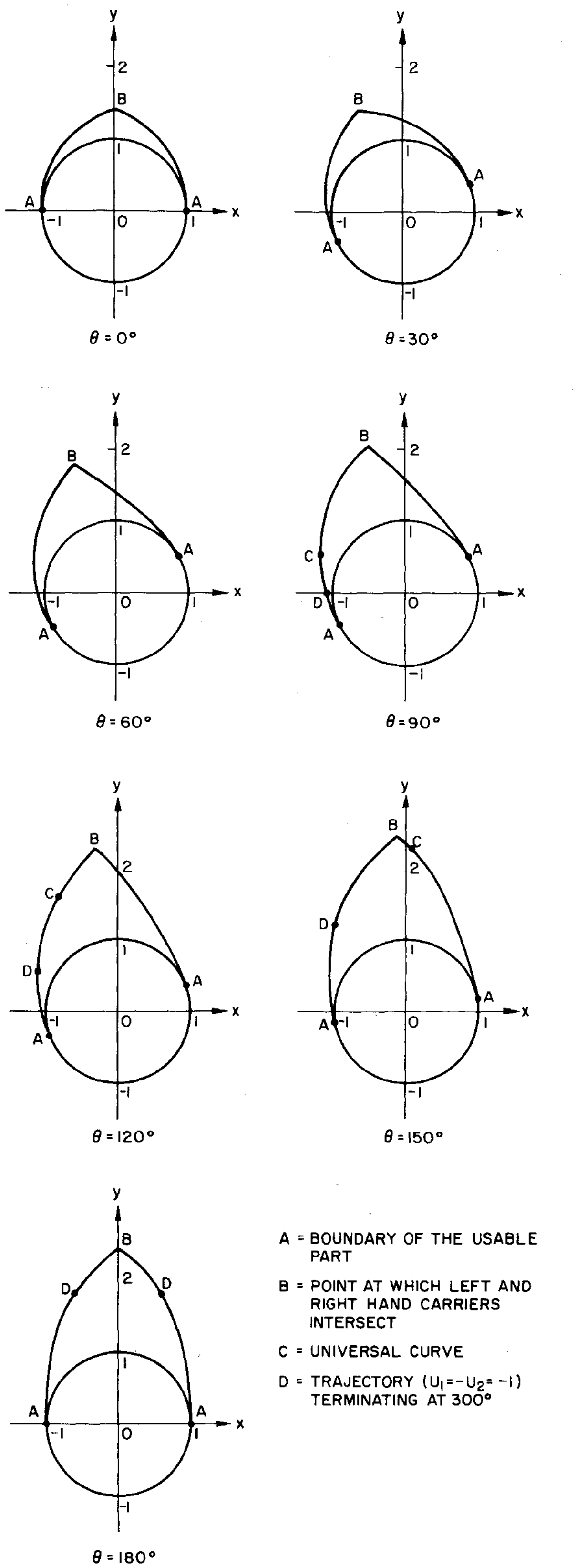

Fig. 8 Cross sections of barrier surface emanating from T_2.

T_1 barrier in the homicidal chauffeur game (Fig. 6) is more extensive than the barrier cross sections in the game of two cars (Fig. 5) (especially for certain relative headings), thus demarcating a larger swerve region there; the player 2 region of capturability in the two-target homicidal chauffeur game (in Fig. 6) is, as would be expected, larger than its counterpart in the game of two cars (see Fig. 5).

VII. Concluding Remarks

We have constructed the regions of win for either player (and draw) in a two-target pursuit-evasion differential game of two cars where the target set for the faster player is a sector of a disk and the target set for the slower player is a complete disk. By employing geometric arguments, we have constructed the complete barrier surfaces in two separate one-target pursuit-evasion games of two cars (thus obtaining, among other things, the region of capturability in a one-target pursuit-evasion game of two cars when the slower player, endowed with a circular target set, is the pursuer) and we then combined these results in order to obtain the regions of capturability in our two-target game of two cars. Although the results are worked out and presented for a specific set of problem parameters, the general procedure is clear.

Acknowledgments

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References

[1] Isaacs, R., *Differential Games,* Wiley, N.Y., 1965, Sec. 9.2.

[2] Blaquiére, A., Gerard, F., and Leitmann, G., *Quantitative and Qualitative Differential Games,* Academic Press, New York, 1965, p. 113.

[3] Getz, W.M. and Leitmann, G., "Qualitative Differential Games with Two Targets," *Journal of Mathematical Analysis and Applications,* Vol. 68, April 1979, pp. 421-430.

[4] Olsder, G.J. and Breakwell, J.V., "Role Determination in Aerial Dogfight," *International Journal of Game Theory,* Vol. 3(1), 1974, pp. 47-66.

[5] Merz, A.W. and Hague, D.S., "Coplanar Tail-Chase Aerial Combat as a Differential Game," *AIAA Journal,* Vol. 15, Oct. 1977, pp. 1419-1423.

[6] Peng, W.Y. and Vincent, T.L., "Some Aspects of Aerial Combat," *AIAA Journal,* Vol. 13, Jan. 1975, pp. 7-11.

[7] Getz, W.M. and Pachter, M., "Two-Target Pursuit-Evasion Differential Games in the Plane," to appear in *Journal of Optimization Theory and Applications,* Vol. 34, No. 3, July 1981.

[8] Pachter, M. and Getz, W.M., "The Geometry of the Barrier in the Game of Two Cars," *Optimal Control, Applications and Methods,* Vol. 1, April-June 1980, pp. 103-118.

[9] Garabedian, P.R., *Partial Differential Equations,* John Wiley, New York, 1964, Chap. 2.